Calculus II

Module Description

Module code	MAT231-F27	**Credits**	7 ECTS
Programme	Foundation programme	**Level**	Semester 3
Semester	Fall 2027	**Prerequisites**	MAT102-S27 Linear Algebra; MAT103-S27 Calculus I
Module leader	Dr. Ali Khudiyev	**Email**	ali.khudiyev@wosm.academy
Office hours	Monday–Friday, 10:00–17:00 (Baku time, UTC+4)		

1. Module Overview

Calculus II continues Calculus I in two directions. First it completes the one-variable theory with infinite series, power series and Taylor series. Then it extends differentiation and integration to functions of several variables, using the vectors and matrices of Linear Algebra.

The module covers:

- Infinite series, convergence tests, power series and Taylor series (first 4–5 weeks)
- Differential calculus of several variables: partial derivatives, gradients, the Jacobian and Hessian, and optimisation with and without constraints
- Multiple integrals and change of variables
- An introduction to ordinary differential equations

These tools are used directly later in the programme: gradients and Hessians in optimisation and machine learning, multiple integrals in Probability Theory and Statistics (joint densities), and series and generating functions in the analysis of algorithms.

2. Aims

- Complete the theory of one-variable calculus with series and uniform convergence.
- Extend differential and integral calculus to functions of several variables.
- Give students the tools for continuous optimisation, including gradient-based methods.
- Prepare students for probability, statistics, machine learning and the analysis of algorithms.

3. Learning Outcomes

On successful completion of this module, students will be able to:

LO1. Decide the convergence of series with the comparison, ratio, root, integral and alternating-series tests, and find the radius of convergence of power series.

LO2. Expand functions as Taylor series, bound the remainder, and use series and generating functions to solve counting and recurrence problems.

LO3. Compute partial derivatives, gradients, directional derivatives, Jacobians and Hessians, and apply the multivariable chain rule.

LO4. Find and classify critical points, solve constrained problems with Lagrange multipliers, and explain and apply gradient descent.

LO5. Evaluate double and triple integrals, change the order of integration, and change variables using polar, cylindrical and spherical coordinates and the Jacobian determinant.

LO6. Solve first-order and linear second-order ordinary differential equations and simple linear systems using eigenvalues.

4. Syllabus

Unit	Topic
1	Infinite series; geometric and telescoping series
2	Convergence tests: comparison, integral, ratio, root
3	Absolute and conditional convergence; alternating series; rearrangements
4	Sequences and series of functions; pointwise and uniform convergence
5	Power series; radius of convergence; term-by-term differentiation and integration
6	Taylor and Maclaurin series; remainder estimates
7	Generating functions; solving recurrences
8	Stirling's approximation
9	$\mathbb{R}^n$: norms, open and closed sets, limits and continuity
10	Partial derivatives; differentiability; the total derivative
11	Gradient and directional derivatives; level sets
12	The multivariable chain rule; the Jacobian matrix
13	Higher derivatives; the Hessian; multivariable Taylor expansion
14	Unconstrained optimisation: critical points, second-derivative test, convexity
15	Gradient descent and its convergence for convex quadratics
16	Constrained optimisation: Lagrange multipliers
17	Implicit and inverse function theorems (statement and use)
18	Double integrals over rectangles and general regions; Fubini's theorem
19	Triple integrals
20	Change of variables; polar, cylindrical and spherical coordinates
21	The Gaussian integral; improper multiple integrals
22	Introduction to vector fields and line integrals
23	First-order ODEs: separable and linear equations
24	Second-order linear ODEs with constant coefficients
25	Systems of linear ODEs via eigenvalues

5. Teaching and Learning Methods

The module is self-paced. All materials are provided on Moodle: lecture notes and recorded lectures for each unit, tutorial problem sheets with worked solutions, short quizzes, and the briefs for the two projects. Problem sheets are for practice and are not marked. Questions are discussed in the module channel on Discord and in office hours. Independent study includes reading, problem sheets and project work.

Activity	Hours
Lectures	50
Labs / Tutorials	30
Independent study	130
Total	**210**

6. Assessment

Knowledge is assessed with weekly unit quizzes, two projects and a final exam. Because much of the module is theory, the exam carries slightly more weight.

- **Project 1 – Series and generating functions:** use generating functions to solve recurrences from counting problems and algorithms, and check the results numerically.
- **Project 2 – Optimisation in several variables:** analyse a function with gradients and Hessians, solve a constrained version with Lagrange multipliers, and implement gradient descent, studying how the step size affects convergence.

The final exam tests computation (series, derivatives and integrals in several variables), the choice of method, and short proofs about convergence.

Component	Weight	Due	LOs
Unit quizzes (best 10 of 12)	20%	Weekly	LO1–LO3, LO5
Project 1: Series and generating functions	15%	Week 6	LO1, LO2
Project 2: Optimisation in several variables	20%	Week 12	LO3, LO4
Final exam (3 hours, written)	45%	End of term	LO1–LO6
Total	**100%**		

7. Reading List

Essential

- G. Strang, E. Herman: *Calculus, Volumes 2 and 3*, OpenStax, 2016. Free online at `https://openstax.org/subjects/math`.
- J. E. Marsden, A. Tromba: *Vector Calculus*, 6th ed., W. H. Freeman, 2012.

Recommended

- T. M. Apostol: *Calculus, Volume 2*, 2nd ed., Wiley, 1969.
- J. H. Hubbard, B. B. Hubbard: *Vector Calculus, Linear Algebra, and Differential Forms*, 5th ed., Matrix Editions, 2015.
- H. S. Wilf: *generatingfunctionology*, 3rd ed., A K Peters, 2006. Free online at `https://www2.math.upenn.edu/~wilf/DownldGF.html`.
- S. Boyd, L. Vandenberghe: *Convex Optimization*, Cambridge University Press, 2004 (Chapter 9).

8. Materials and Licence

Course materials are published under the Creative Commons Attribution-ShareAlike 4.0 licence unless stated otherwise. See wosm.academy/licence for details.

9. Policies

- **Attendance:** Attendance is not mandatory. The module is self-paced, so you can study the materials when it suits you.
- **Late submissions:** Work submitted after the deadline is penalised.
- **Academic integrity:** All submitted work must be your own. See the Code of Conduct.