Calculus I

Module Description

Module code	MAT103-S27	**Credits**	7 ECTS
Programme	Foundation programme	**Level**	Semester 2
Semester	Spring 2027	**Prerequisites**	MAT101-F26 Discrete Structures
Module leader	Dr. Ali Khudiyev	**Email**	ali.khudiyev@wosm.academy
Office hours	Monday–Friday, 10:00–17:00 (Baku time, UTC+4)		

1. Module Overview

Calculus I is the first module of analysis in the curriculum. It builds the calculus of functions of one real variable on a precise footing: the real numbers, limits and continuity come first, and differentiation and integration are then developed from them rather than taken as a set of rules.

The module covers:

- The real numbers, sequences and limits (first 4–5 weeks)
- Continuity, differentiation and its applications: optimisation, approximation and Newton's method
- The Riemann integral, the Fundamental Theorem of Calculus and techniques of integration

Throughout, the module links the theory to computer science: convergence of iterative algorithms, asymptotic growth of functions, numerical approximation and floating-point error. It prepares students for Calculus II and for Probability Theory and Statistics.

2. Aims

- Give a rigorous understanding of limits, continuity, derivatives and integrals of functions of one real variable.
- Develop fluency in computing derivatives and integrals and in using them to model and solve problems.
- Train students to read and write short, correct proofs in analysis using ε–δ arguments.
- Show how calculus underpins algorithms and numerical methods used in computer science.

3. Learning Outcomes

On successful completion of this module, students will be able to:

LO1. State the completeness property of the real numbers and use it to reason about suprema, infima and convergence of sequences.

LO2. Compute limits of sequences and functions, and prove simple limit and continuity statements with ε–δ arguments.

LO3. Differentiate elementary functions and apply the Mean Value Theorem, L'Hôpital's rule and Taylor polynomials to analyse and approximate functions.

LO4. Solve optimisation and related-rates problems and use Newton's method and bisection to approximate roots, estimating the error.

LO5. Evaluate definite and indefinite integrals using the Fundamental Theorem of Calculus, substitution, integration by parts and partial fractions.

LO6. Use integrals to compute areas, volumes and averages, and compare the growth of functions as used in asymptotic analysis of algorithms.

4. Syllabus

Unit	Topic
1	The real numbers: ordered fields, absolute value, inequalities
2	Completeness: supremum, infimum, the Archimedean property
3	Sequences: convergence, limit laws, bounded and monotone sequences
4	Subsequences, the Bolzano–Weierstrass theorem, Cauchy sequences
5	Functions: elementary functions, inverses, exponential and logarithm
6	Limits of functions: ε–δ definition, one-sided limits
7	Continuity and its algebra
8	Intermediate Value Theorem, Extreme Value Theorem; the bisection method
9	Uniform continuity
10	The derivative: definition, rules, chain rule
11	Derivatives of inverse, exponential, logarithmic and trigonometric functions
12	Mean Value Theorem, Rolle's theorem, monotonicity
13	L'Hôpital's rule; growth rates and comparison with O-notation
14	Higher derivatives, convexity, curve sketching
15	Optimisation and related rates
16	Newton's method and its convergence
17	Taylor polynomials and Taylor's theorem with remainder
18	The Riemann integral: partitions, upper and lower sums
19	Integrability of continuous and monotone functions; properties of the integral
20	The Fundamental Theorem of Calculus
21	Integration by substitution and by parts
22	Partial fractions and trigonometric integrals
23	Improper integrals
24	Applications: area, volume, arc length, average value
25	Numerical integration: trapezoidal and Simpson's rules, error bounds
26	Sums and integrals: bounding $\sum f(k)$ by $\int f$, harmonic numbers

5. Teaching and Learning Methods

The module is self-paced. All materials are provided on Moodle: lecture notes and recorded lectures for each unit, tutorial problem sheets with worked solutions, short quizzes, and the briefs for the two projects. Problem sheets are for practice and are not marked. Questions are discussed in the module channel on Discord and in office hours. Independent study includes reading, problem sheets and project work.

Activity	Hours
Lectures	52
Labs / Tutorials	28
Independent study	130
Total	**210**

6. Assessment

Knowledge is assessed with weekly unit quizzes, two projects and a final exam. Quizzes check computational fluency. The projects combine short proofs, calculation and a small amount of code:

- **Project 1 – Roots and approximation:** implement bisection and Newton's method, compare their convergence, and use Taylor polynomials to approximate functions with proven error bounds.
- **Project 2 – Integrals and asymptotics:** implement the trapezoidal and Simpson's rules with error bounds, and use integrals to bound sums such as harmonic numbers that appear in the analysis of algorithms.

The final exam tests whether students can choose and apply the right technique and justify each step.

Component	Weight	Due	LOs
Unit quizzes (best 10 of 12)	20%	Weekly	LO1–LO3, LO5
Project 1: Roots and approximation	20%	Week 7	LO2–LO4
Project 2: Integrals and asymptotics	20%	Week 12	LO5, LO6
Final exam (3 hours, written)	40%	End of term	LO1–LO6
Total	**100%**		

7. Reading List

Essential

- M. Spivak: *Calculus*, 4th ed., Publish or Perish, 2008.
- G. Strang, E. Herman: *Calculus, Volume 1*, OpenStax, 2016. Free online at `https://openstax.org/details/books/calculus-volume-1`.

Recommended

- J. Stewart, D. Clegg, S. Watson: *Calculus: Early Transcendentals*, 9th ed., Cengage, 2020.
- T. M. Apostol: *Calculus, Volume 1*, 2nd ed., Wiley, 1967.
- S. Abbott: *Understanding Analysis*, 2nd ed., Springer, 2015.
- R. L. Graham, D. E. Knuth, O. Patashnik: *Concrete Mathematics*, 2nd ed., Addison-Wesley, 1994 (Chapter 9, asymptotics).

8. Materials and Licence

Course materials are published under the Creative Commons Attribution-ShareAlike 4.0 licence unless stated otherwise. See wosm.academy/licence for details.

9. Policies

- **Attendance:** Attendance is not mandatory. The module is self-paced, so you can study the materials when it suits you.
- **Late submissions:** Work submitted after the deadline is penalised.
- **Academic integrity:** All submitted work must be your own. See the Code of Conduct.