Linear Algebra

Module Description

Module code	MAT102-S27	**Credits**	7 ECTS
Programme	Foundation programme	**Level**	Semester 2
Semester	Spring 2027	**Prerequisites**	MAT101-F26 Discrete Structures
Module leader	Dr. Ali Khudiyev	**Email**	ali.khudiyev@wosm.academy
Office hours	Monday–Friday, 10:00–17:00 (Baku time, UTC+4)		

1. Module Overview

Linear Algebra is the study of vectors, linear maps and the matrices that represent them. It is one of the most widely used areas of mathematics in computer science: graphics, machine learning, data analysis, search engines, coding theory and scientific computing all rest on it.

The module covers:

- Systems of linear equations, Gaussian elimination and matrix algebra (first 4–5 weeks)
- Vector spaces, linear maps, bases and dimension, determinants
- Inner products, orthogonality and least squares
- Eigenvalues, diagonalisation, the spectral theorem and the singular value decomposition

The module balances the abstract view (vector spaces and linear maps) with computation (matrices and algorithms). Students implement the core algorithms in short programming exercises and meet applications such as PageRank, image compression and linear regression. It builds on the sets, functions and algebraic structures of Discrete Structures and is required for Probability Theory and Statistics.

2. Aims

- Introduce vector spaces and linear maps as the common language for many problems in computing and science.
- Develop fluency with matrix computations and the algorithms behind them, including their cost and numerical behaviour.
- Build geometric intuition for subspaces, projections, orthogonality and eigenvectors.
- Prepare students for probability, statistics, machine learning, graphics and numerical methods.

3. Learning Outcomes

On successful completion of this module, students will be able to:

LO1. Solve systems of linear equations with Gaussian elimination and LU factorisation, and describe the solution set in terms of rank and null space.

LO2. Work with abstract vector spaces: decide linear independence, find bases and dimensions, and apply the rank–nullity theorem.

LO3. Represent linear maps by matrices, change basis, and compute and interpret determinants.

LO4. Use inner products to compute orthogonal projections, apply Gram–Schmidt and QR factorisation, and solve least-squares problems.

LO5. Compute eigenvalues and eigenvectors, diagonalise matrices where possible, and apply the spectral theorem to symmetric matrices.

LO6. Compute and interpret the singular value decomposition and apply it to low-rank approximation and data analysis.

LO7. Implement core linear algebra algorithms in code and estimate their computational cost.

4. Syllabus

Unit	Topic
1	Vectors in $\mathbb{R}^n$: linear combinations, dot product, length, angle
2	Systems of linear equations; Gaussian and Gauss–Jordan elimination
3	Matrix operations; matrix multiplication as composition
4	Inverse matrices; elementary matrices; LU factorisation
5	Cost of elimination; pivoting and numerical stability
6	Vector spaces and subspaces; fields beyond $\mathbb{R}$ (e.g. $\mathbb{F}_2$)
7	Span, linear independence, basis, dimension
8	The four fundamental subspaces; rank; rank–nullity theorem
9	Linear maps: kernel, image, injectivity and surjectivity
10	Matrix of a linear map; change of basis; similarity
11	Determinants: definition, properties, cofactor expansion
12	Determinants: volume, invertibility, Cramer's rule
13	Inner product spaces; norms; Cauchy–Schwarz inequality
14	Orthogonality; orthogonal complements; orthogonal projections
15	Gram–Schmidt process; QR factorisation
16	Least squares and linear regression
17	Eigenvalues and eigenvectors; characteristic polynomial
18	Diagonalisation; powers of matrices; linear recurrences (Fibonacci)
19	Markov chains and PageRank
20	Symmetric matrices; the spectral theorem
21	Quadratic forms; positive definite matrices
22	Singular value decomposition
23	Low-rank approximation; principal component analysis
24	Iterative methods: power iteration; introduction to Jordan form
25	Applications: computer graphics transforms, linear codes over $\mathbb{F}_2$

5. Teaching and Learning Methods

The module is self-paced. All materials are provided on Moodle: lecture notes and recorded lectures for each unit, tutorial problem sheets with worked solutions, short quizzes, and the briefs for the two projects. Problem sheets are for practice and are not marked. Questions are discussed in the module channel on Discord and in office hours. Independent study includes reading, problem sheets and project work.

Activity	Hours
Lectures	50
Labs / Tutorials	40
Independent study	120
Total	**210**

6. Assessment

Knowledge is assessed with weekly unit quizzes, two programming projects and a final exam. Labs are used for the programming work.

- **Project 1 – Solving linear systems:** implement Gaussian elimination with partial pivoting and LU factorisation, measure cost and numerical error, and fit a model to data by least squares.
- **Project 2 – Eigenvectors and the SVD:** rank web pages with PageRank by power iteration, and compress images with a truncated SVD, explaining the trade-off between rank and error.

The final exam tests the standard computations, the choice of method for a problem and short proofs about vector spaces and linear maps.

Component	Weight	Due	LOs
Unit quizzes (best 10 of 12)	20%	Weekly	LO1–LO6
Project 1: Solving linear systems	20%	Week 6	LO1, LO4, LO7
Project 2: Eigenvectors and the SVD	20%	Week 12	LO5–LO7
Final exam (3 hours, written)	40%	End of term	LO1–LO6
Total	**100%**		

7. Reading List

Essential

- G. Strang: *Introduction to Linear Algebra*, 6th ed., Wellesley-Cambridge Press, 2023.
- J. Hefferon: *Linear Algebra*, 4th ed., 2020. Free online at hefferon.net/linearalgebra.

Recommended

- S. Axler: *Linear Algebra Done Right*, 4th ed., Springer, 2024 (open access).
- S. Boyd, L. Vandenberghe: *Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares*, Cambridge University Press, 2018.
- L. N. Trefethen, D. Bau: *Numerical Linear Algebra*, SIAM, 1997.

8. Materials and Licence

Course materials are published under the Creative Commons Attribution-ShareAlike 4.0 licence unless stated otherwise. See wosm.academy/licence for details.

9. Policies

- **Attendance:** Attendance is not mandatory. The module is self-paced, so you can study the materials when it suits you.
- **Late submissions:** Work submitted after the deadline is penalised.
- **Academic integrity:** All submitted work must be your own. See the Code of Conduct.